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Again, the normal at P to the path of P , $\rho = f(\theta)$, must also pass through C . Hence the angle OPC is the complement of the angle ψ between OP and the tangent at P . Hence,

$$\tan OPC = \cot \psi = \frac{d\rho}{\rho d\theta}.$$

(See any book on Calculus.) Therefore, from the right triangle COP , we have

$$OC = \rho \tan OPC = \frac{d\rho}{d\theta} = \frac{d}{d\theta} f(\theta).$$

Also solved by J. B. REYNOLDS and W. E. CEDERBERG.

257 (Number Theory). Proposed by LOUIS O'SHAUGHNESSY, University of Pennsylvania.

Find a general expression for the number of positive integers from 1 to 10^t , inclusive, every one of which contains the figure 9 exactly r times ($0 \leq r \leq t$).

SOLUTION BY THE PROPOSER.

In the case of the integers from 1 to 10, we have nine which do not contain the figure 9 and one which contains one 9. This shall be indicated by the expression $9 + 1$.

In the case of 10^2 , the number of integers, which do not contain 9, is $9 \cdot 9$, or 9^2 ; which contain one 9, is $9 \cdot 1 + 9$, or $2 \cdot 9$; which contain two 9's, is 1, and we have the expansion of

$$(9 + 1)^2 = 9^2 + 2 \cdot 9 + 1.$$

For 10^3 , we have $9 \cdot 9^2$, $9 \cdot 2 \cdot 9 + 9^2$, $9 \cdot 1 + 2 \cdot 9$, and 1, or $9^3 + 3 \cdot 9^2 + 3 \cdot 9 + 1$.

Then, for 10^k , assume the expansion of $(9 + 1)^k$, or

$$9^k + \binom{k}{1} 9^{k-1} + \binom{k}{2} 9^{k-2} + \dots + \binom{k}{n-1} 9^{k-(n-1)} + \binom{k}{n} 9^{k-n} + \dots + \binom{k}{k-1} 9 + 1.$$

For 10^{k+1} we reason as follows: The number of integers which do not contain 9 is $9 \cdot 9^k$, or 9^{k+1} ; which contain one 9, is $9 \cdot \binom{k}{1} 9^{k-1} + 9^k$, or $\binom{k+1}{1} 9^k$; which contain two 9's, is $9 \cdot \binom{k}{2} 9^{k-2} + \binom{k}{1} 9^{k-1}$ or $\binom{k+1}{2} 9^{k-1}$, and which contain n 9's, is

$$9 \cdot \binom{k}{n} 9^{k-n} + \binom{k}{n-1} 9^{k-(n-1)} = [\binom{k}{n} + \binom{k}{n-1}] 9^{k-n+1} = \binom{k+1}{n} 9^{k+1-n}.$$

Hence, we have, for 10^{k+1} , the expansion of $(9 + 1)^{k+1}$, or

$$9^{k+1} + \binom{k+1}{1} 9^k + \dots + \binom{k+1}{n} 9^{k+1-n} + \dots + \binom{k+1}{k} 9 + 1.$$

Now, the derived expression holds for $k = 2$ and for $k = 3$; hence it holds for all positive integral values of k .

Therefore, the general expression required is $\binom{t}{r} 9^{t-r}$.

Also solved by HORACE OLSON, H. C. FEEMSTER, C. C. YEN, and N. P. PANDYA.

258 (Number Theory). Proposed by A. A. BENNETT, University of Texas.

Find a recursion formula in terms of binomial coefficients for a_n , where the a 's are defined by the condition that the persymmetric determinants

$$\begin{vmatrix} a_0 & a_1 & a_2 & \cdot & \cdot \\ a_1 & a_2 & \cdot & \cdot & \cdot \\ a_2 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & a_{n-1} \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} a_1 & a_2 & a_3 & \cdot & \cdot \\ a_2 & a_3 & \cdot & \cdot & \cdot \\ a_3 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & a_n \end{vmatrix}$$

are each equal to unity for every positive integer n .

SOLUTION BY C. F. GUMMER, Queen's University, Kingston, Ont.

Though this solution does not directly involve binomial coefficients, yet by finding the value of a_n it may be considered to dispose of the problem sufficiently.